Lab 2 - Mathematical Programming Examples

Eren Darici M.S.
eren.darc@wmich.edu

WMU - IEE3110

February 6, 2026

Today's Agenda

- Cake Production
- Diet Problem
- Dorian Auto
  - Open form
  - Closed form

Cake Production

Plain cake requires **200g of flour and 25g of fat**, and chocolate cake requires **100g of flour and 50g of fat**.
Formulate the decision model to **find the maximum number of cakes** that can be made from **5kg of flour and 1 kg of fat** assuming that there is no shortage of the other ingredients used in making the cakes.

Cake Production - Model

Decision variables:
x_1 : number of plain cakes produced
x_2 : number of chocolate cakes produced
Maximize

$$\max Z = x_1 + x_2$$

Subject to

$$\begin{aligned} 200x_1 + 100x_2 &\leq 5000 \quad \text{(flour constraint)} \\ 25x_1 + 50x_2 &\leq 1000 \quad \text{(fat constraint)} \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

$$x_1, x_2 \in \mathbb{Z}_{\geq 0}$$

Diet Problem

My diet requires that all the food I get come from one of the four "basic food groups". At present, the following four foods are available for consumption: brownies, chocolate ice cream, juice and pineapple cheesecake costs 80 ¢. cheesecake.

- Each brownie costs 50¢, each scoop of ice cream costs 20 ¢, each bottle of juice costs 30 ¢, and each piece of pineapple
- Each day, I must ingest at least 500 calories, 6 oz of chocolate, 10 oz of sugar, and 8 oz of fat.

The nutritional content per unit of each food is given in the next slide.
Formulate a decision model that can be used to satisfy my daily nutritional requirements at minimum cost.

Diet Problem - cont.

Item	Calories	Chocolate (oz)	Sugar (oz)	Fat (oz)
Brownie	400	3	2	2
Chocolate Ice Cream (1 scoop)	200	2	2	4
Juice (1 bottle)	250	0	4	1
Pineapple Cheesecake (1 piece)	500	0	4	5

Table: Nutritional information for selected food items

Diet Problem - Closed Form

Sets

$J = \{1, 2, 3, 4\}$ set of food items

$I = \{1, 2, 3, 4\}$ set of requirements

Decision Variables

x_j : Amount of food j consumed

$$x_j \geq 0 \qquad \forall j \in J$$

Parameters

c_j	cost per unit of item j
a_{ij}	contribution of item j to requirement i
b_i	minimum required level of requirement i

Diet Problem - Closed Form

$$
\begin{aligned}
\min \quad & Z = \sum_{j \in J} c_j x_j & \\
\text{s.t.} \quad & \sum_{j \in J} a_{ij} x_j \geq b_i & \forall i \in I \\
& x_j \geq 0 & \forall j \in J
\end{aligned}
$$

Diet Problem - Closed Form

Data

$$\mathbf{c} = \begin{bmatrix} 0.50 & 0.20 & 0.30 & 0.80 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} 500 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 400 & 200 & 250 & 500 \\ 3 & 2 & 0 & 0 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 1 & 5 \end{bmatrix}$$

PuLP: `LpVariable.dicts`

Purpose: `LpVariable.dicts` is used to create a collection of decision variables indexed over one or more sets (similar to indexed variables in math models).

Syntax:

```
LpVariable.dicts(name, index_set, lowBound, upBound, cat)
```

Example:

```
x = pulp.LpVariable.dicts( "x", foods, lowBound=0, cat="Continuous" )
```

This creates variables:

$$x_1, x_2, x_3, x_4 \geq 0$$

which can be used in objectives and constraints via indexing (e.g., `x[j]`).

PuLP: `lpSum`

Purpose:

`lpSum` is used to efficiently construct linear expressions involving summations of decision variables (analogous to $\sum$ notation in math).

Syntax:

```
lpSum(expression for index in set)
```

- Takes a Python generator or list of linear terms
- Returns a single linear expression
- Preferred over Python `sum()` for performance and clarity

Example:

```
model += pulp.lpSum(c[j] * x[j] for j in foods)
```

Mathematical equivalent:

$$\min \; Z = \sum_{j \in J} c_j x_j$$

Dorian Auto

Dorian Auto manufactures luxury cars and trucks. The company believes that its most likely customers are high-income women and men.
To reach these groups, Dorian Auto has embarked on an ambitious TV advertising campaign and will purchase 1-minute commercial spots on two type of programs: comedy shows and football games.

- Each comedy commercial is seen by 7 million high income women and 2 million high-income men and costs $50,000$.
- Each football game is seen by 2 million high-income women and 12 million high-income men and costs $100,000$.
- Dorian Auto would like for commercials to be seen by at least 28 million high-income women and 24 million high-income men.

How can Dorian Auto meet its advertising requirements at **minimum cost**?

Dorian Auto - Open Form

Decision Variables:

$$x_1 := \text{number of 1-minute commercials purchased on comedy shows}$$
$$x_2 := \text{number of 1-minute commercials purchased during football games}$$

Minimize Cost

$$\min \; Z = 50{,}000\, x_1 + 100{,}000\, x_2$$

Subject to:

$$\begin{aligned} 7x_1 + 2x_2 &\geq 28 \quad \text{(high-income women, in millions)} \\ 2x_1 + 12x_2 &\geq 24 \quad \text{(high-income men, in millions)} \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

Dorian Auto - Closed Form

Sets

$J = \{1, 2\}$ set of advertising options

$I = \{1, 2\}$ set of audience groups

Decision Variables

x_j : Amount of minutes bought for commercial type j

$$x_j \geq 0 \qquad \forall j \in J$$

Parameters

c_j	cost of one commercial of type j
a_{ij}	reach (in millions) of audience group i per ad j
b_i	minimum required reach for audience group i

Dorian Auto - Closed Form

$$
\begin{aligned}
\min \quad & Z = \sum_{j \in J} c_j x_j \\
\text{s.t.} \quad & \sum_{j \in J} a_{ij} x_j \geq b_i & \forall i \in I \\
& x_j \geq 0 & \forall j \in J
\end{aligned}
$$

Dorian Auto - Closed Form

Data

$$\mathbf{c} = \begin{bmatrix} 50{,}000 & 100{,}000 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} 28 \\ 24 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 7 & 2 \\ 2 & 12 \end{bmatrix}$$