# Lab 3 - Assignment and Transportation Problems

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# Today's Agenda

- Recap: Assignment and Transportation Problems
- Powerplant Example
- PuLP LpVariable.dicts
- Useful Pandas Functions for Large Scale Optimization

# Recap: Transportation & Assignment Problems

- The **Assignment Problem** is a special case of the **Transportation Problem**.
- In assignment problems, most of the technical coefficients ($a_{ij}$) are zero, which allows for the use of more efficient, specialized algorithms.
- A transportation problem is a **many-to-many (m:n) matching problem** between two distinct sets, making it a **bipartite problem**.
- A common example involves **factories (sources)** and **regions (destinations)**.
- The objective is to determine how much product should be transported from each factory to each region in order to **minimize total transportation cost**.

*Source: Dr. Acar's slides*

# Transportation Problem: Closed-Form Model

**Decision Variables:**

$$x_{ij} = \text{amount shipped from source } i \text{ to destination } j$$

**Parameters:**

- $c_{ij}$: unit transportation cost from $i$ to $j$
- $s_i$: supply capacity at source $i$
- $d_j$: demand requirement at destination $j$

# Transportation Problem: Closed Form Model

**Objective Function:**

$$\min \sum_i \sum_j c_{ij} x_{ij}$$

**Constraints:**

- **Supply constraints:**

$$\sum_j x_{ij} \leq s_i \quad \forall i$$

- **Demand constraints:**

$$\sum_i x_{ij} \geq d_j \quad \forall j$$

- **Non-negativity:**

$$x_{ij} \geq 0 \quad \forall i, j$$

# Transportation Problem

Powerco operates three electric power plants that supply electricity to four cities. The supply capacities of the plants (in million kWh) are as follows:

- Plant 1: 35
- Plant 2: 50
- Plant 3: 40

The peak power demands of the cities (in million kWh) are:

- City 1: 45
- City 2: 20
- City 3: 30
- City 4: 30

The cost of transmitting 1 million kWh of electricity from each plant to each city is provided in the table below.

**Task:** Formulate this as a balanced transportation problem using a transportation tableau and express it as a linear programming (LP) model.

# Transportation Tableau

| **Plant** | **City 1** | **City 2** | **City 3** | **City 4** | **Supply** |
|---|---|---|---|---|---|
| Plant 1 | \$8 | \$6 | \$10 | \$9 | 35 |
| Plant 2 | \$9 | \$12 | \$13 | \$7 | 50 |
| Plant 3 | \$14 | \$9 | \$16 | \$5 | 40 |
| **Demand** | 45 | 20 | 30 | 30 | 125 |

(All costs are per 1 million kWh; supply and demand are in million kWh.)

# Balanced Transportation Problem (Open LP Form)

**Decision Variables:**

$$x_{ij} = \text{amount of electricity shipped from Plant } i \text{ to City } j$$

**Objective Function:**

$$\begin{aligned} \min Z = {} & 8x_{11} + 6x_{12} + 10x_{13} + 9x_{14} \\ & + 9x_{21} + 12x_{22} + 13x_{23} + 7x_{24} \\ & + 14x_{31} + 9x_{32} + 16x_{33} + 5x_{34} \end{aligned}$$

# Balanced Transportation Problem (Open LP Form)

**Subject to Supply Constraints:**

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 35$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 50$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 40$$

**Subject to Demand Constraints:**

$$x_{11} + x_{21} + x_{31} \geq 45$$

$$x_{12} + x_{22} + x_{32} \geq 20$$

$$x_{13} + x_{23} + x_{33} \geq 30$$

$$x_{14} + x_{24} + x_{34} = 30$$

**Non-negativity:**

$$x_{ij} \geq 0 \quad \forall i, j$$

# Decision Variables in PuLP: `LpVariable.dicts`

**Purpose**

- Used to define **multiple decision variables at once**.
- Creates a structured collection of variables indexed by sets (e.g., plants and cities).

**Idea**

- In transportation problems, we need one variable for each route:

$$x_{ij} \quad \text{(shipment from source } i \text{ to destination } j\text{)}$$

- Instead of defining them one-by-one, we group them in a **dictionary structure**.

**Why It Is Useful**

- Makes large-scale models compact and readable.
- Matches the mathematical structure of optimization models.
- Allows easy indexing like $x_{11}, x_{12}, x_{23}, \ldots$

# Pandas Concepts for Data Handling

**CSV (Comma-Separated Values)**

- A simple file format used to store tabular data (like Excel sheets).
- Each row represents an observation, and each column represents a variable.
- Common format for importing/exporting data in optimization problems.

**DataFrame**

- A DataFrame is a **2-dimensional data structure** in Pandas.
- It organizes data into labeled rows and columns.
- It allows efficient storage, access, and manipulation of structured data.
- Equivalent to a spreadsheet or a database table.

**Index-based Selection (iloc)**

- A method used to select data based on **position (row and column numbers)**.
- Works like matrix indexing in mathematics.
- Useful when the structure of the dataset is known and fixed.