Lab 4 - Assignment Problems and Shortest Path

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Today's Agenda

- Assignment Problem Example - Machineco
- Shortest Path Mathematical Model
- Shortest Path Example - Seervada

Assignment Problem - Machineco (Problem Definition)

Problem Description:
Machineco has four machines and four jobs to be completed. Each machine must be assigned to exactly one job, and each job must be assigned to exactly one machine.

The setup times (in hours) required for assigning machines to jobs are given below:

	Job 1	Job 2	Job 3	Job 4
Machine 1	14	5	8	7
Machine 2	2	12	6	5
Machine 3	7	8	3	9
Machine 4	2	4	6	10

Objective: Minimize the total setup time required to complete all jobs.

Assignment Problem - Mathematical Model

Decision Variables:

$$x_{ij} = \begin{cases} 1 & \text{if machine } i \text{ is assigned to job } j \\ 0 & \text{otherwise} \end{cases}$$

Objective Function:

$$\min Z = \sum_{i=1}^{4} \sum_{j=1}^{4} c_{ij} x_{ij}$$

Constraints:

(Each machine is assigned to exactly one job)

$$\sum_{j=1}^{4} x_{ij} = 1 \quad \forall i = 1, 2, 3, 4$$

(Each job is assigned to exactly one machine)

Shortest Path Problem - General Model

Problem Description:
Given a network with nodes and arcs (edges), each arc has an associated cost (distance, time, etc.). The goal is to find the minimum-cost path from a source node to a destination node.

Decision Variables:

$$x_{ij} = \begin{cases} 1 & \text{if arc } (i,j) \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

Objective Function:

$$\min Z = \sum_{(i,j)\in A} c_{ij} x_{ij}$$

Shortest Path Problem - General Model

Flow Conservation Constraints:

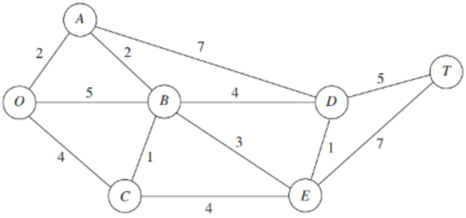
$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = \begin{cases} 1 & \text{if } i = \text{source} \\ -1 & \text{if } i = \text{destination} \\ 0 & \text{otherwise} \end{cases}$$

Binary Constraints:

$$x_{ij} \in \{0, 1\}$$

Shortest Path Example - Seerveda

Problem Description:
Seerveda Park wants to determine the shortest path from the origin node O to the destination node T.



Each arc represents a possible path with an associated distance.

Goal: Find the minimum-distance path from node O (origin) to node T (destination).

Seerveda Example - Mathematical Model

Decision Variables:

$$x_{ij} = \begin{cases} 1 & \text{if arc } (i, j) \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

Objective Function:

$$\min Z = \sum c_{ij} x_{ij}$$

Seerveda Example - Mathematical Model

Flow Constraints:

Origin node (O):

$$\sum_j x_{Oj} - \sum_j x_{jO} = 1$$

Destination node (T):

$$\sum_j x_{jT} - \sum_j x_{Tj} = 1$$

Intermediate nodes (A, B, C, D, E):

$$\sum_j x_{ij} - \sum_j x_{ji} = 0$$

Binary Constraints:

$$x_{ij} \in \{0, 1\}$$